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A small initial data criterion of global existence for the damped nonlinear Schrödinger equation

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Abstract
This paper is concerned with the damped nonlinear Schrödinger equation. Through analyzing the characteristics of the equation and the effect of the damping on the global existence, we construct a variational problem. Then combining the variational problem, we establish a crucial invariant evolution flow to derive an explicit and computed criterion to answer: how small are the initial data such that the solutions of the system globally exist? Moreover, the small initial data criterion can be applied in the nonlinear Schrödinger equation with any positive damped parameter.

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1. Introduction

The nonlinear Schrödinger equation

$$\mathrm{i}\varphi_t + \triangle\varphi + |\varphi|^{p-1}\varphi = 0, \qquad \varphi \in H^1(\mathbf{R}^N) \tag{1.1}$$

is one of the basic evolution models for nonlinear waves in various branches of physics. It can describe the propagation of light beams with Kerr nonlinearity in nonlinear optics [8]; it also occurs in hydrodynamics [20] and plasma physics [14]. However, almost all these applications are under 'ideal transparency' by neglecting the effect of damping [5].

In this paper we are interested in equation (1.1) with a linear damped term as follows:

$$\mathrm{i}\varphi_t + \triangle\varphi + \mathrm{i}a\varphi + |\varphi|^{p-1}\varphi = 0. \tag{1.2}$$

Here $\varphi = \varphi(x, t) : \mathbf{R}^N \times [0, T_a) \longrightarrow \mathbf{C}$ is a complex valued wavefunction, and $0 < T_a \leqslant +\infty$ is the maximal existence time. N is the space dimension, $\mathrm{i} = \sqrt{-1}$, $a > 0$ is the damped parameter, $\triangle$ is the Laplace operator on $\mathbf{R}^N$ and the nonlinear power exponent p satisfies

$1 < p < \frac{N+2}{(N-2)^+}$ (we use the convention: $\frac{N+2}{(N-2)^+} = +\infty$ when $N = 1, 2$, and $\frac{N+2}{(N-2)^+} = \frac{N+2}{N-2}$ when $N \geqslant 3$). Equation (1.2) arises in various areas of nonlinear optics, plasma physics and fluid mechanics [1, 2, 5, 6, 10, 19]. For equation (1.2), impose the initial datum

$$\varphi(x, 0) = \varphi_0, \qquad x \in \mathbf{R}^N. \tag{1.3}$$

Then equations (1.2) and (1.3) form a Cauchy problem. It follows from Kato [7], Cazenave [4] and Tsutsumi [15, 16] that the local well posedness of the Cauchy problem (1.2)–(1.3) in $H^1(\mathbf{R}^N)$ is as follows.

For $1 < p < \frac{N+2}{(N-2)^+}$, let the initial datum $\varphi_0 \in H^1(\mathbf{R}^N)$. Then there exists a unique solution $\varphi(x, t) \in C([0, T_a); H^1(\mathbf{R}^N))$ of the Cauchy problem (1.2)–(1.3). Here $T_a \in [0, \infty]$ is the maximal existence time such that T satisfies the alternative: either $T_a = \infty$ (global existence), or $T_a < \infty$ and $\lim_{t\to T_a} \|\varphi(x, t)\|_{H^1} = \infty$ (blowup).

In particular, equation (1.2) is heavily different from equation (1.1), which is one of the motivations for us to consider equation (1.2). One important reason is that equation (1.2) loses the conservation laws of mass and energy, which, however, are possessed by equation (1.1) [4]. The other important reason is that equation (1.2) cannot have any stationary or time-periodic solutions because the mass is attenuating described by (2.6), which, however, is possessed by equation (1.1) [4].

For equation (1.2), when $1 < p < 1 + \frac{4}{N}$, from Ohta and Todorova [12], Tsustumi [17] and Cazenave [4], we know that the solutions of the Cauchy problem (1.2)–(1.3) globally exist for all initial data. When $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$, from Tsutsumi [15], Fibich [5], Ohta and Todorova [12], we know that the solutions of the Cauchy problem (1.2)–(1.3) can blow up in a finite time for some initial data, especially for some sufficiently large initial data; but the solutions of the Cauchy problem (1.2)–(1.3) also can globally exist in $H^1(\mathbf{R}^N)$ for some initial data, especially for some sufficiently small initial data. Therefore there naturally appears a problem as follows.

Problem 1.1. *How small are the initial data in $H^1(\mathbf{R}^N)$ such that the solutions of the Cauchy problem (1.2)–(1.3) globally exist for $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$?*

In this paper, we are especially concerned with problem 1.1. Our idea is originated in Tsutsumi [15, 16], Fibich [5], Ohta and Todorova [12] and Levine [11]. First, we fix $1+\frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$. Through analyzing the characteristics of the Cauchy problem (1.2)–(1.3) and the effect of the damping on the global existence, we construct a variational problem. Then combining the variational problem, we establish a crucial invariant evolution flow. At last, we derive an explicit and computed criterion to answer problem 1.1 as follows. First, let $Q(x)$ be the positive and spherically symmetric solution of the nonlinear elliptic equation

$$-\Delta u + u - |u|^{p-1}u = 0, \qquad u \in H^1(\mathbf{R}^N). \tag{1.4}$$

Strauss [13] got the existence of solutions of equation (1.4), and Kwong [9] proved the uniqueness of the solutions of equation (1.4).

Theorem 1.1 (small initial data criterion). *For $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$, let $Q \in H^1(\mathbf{R}^N)$ be the positive and spherically symmetric solution of equation (1.4). Assume the damped parameter $a > 0$. If the initial datum $\varphi_0 \in H^1(\mathbf{R}^N)$ satisfies*

$$\|\varphi_0\|_{H^1} \leqslant \sqrt{\frac{2(p-1)}{2(p+1) - N(p-1)}}\|Q\|_{L^2},$$

then the solution $\varphi(t)$ of the Cauchy problem (1.2)–(1.3) globally exists in $H^1(\mathbf{R}^N)$.

Remark 1.1. Especially, consider the case of $N = 2$ and $p = 3$. We know that $\|Q\|_2^2 \cong 11.700\,86$ for equation (1.4) (see Weinstein [18]). Therefore, for this case, if the initial datum $\varphi_0 \in H^1(\mathbf{R}^N)$ satisfies $\|\varphi_0\|_{H^1} \leqslant 3.42065$, the solution $\varphi(t)$ of the Cauchy problem (1.2)–(1.3) globally exists in $H^1(\mathbf{R}^2)$.

Remark 1.2. It is interesting in that the small initial data criterion (theorem 1.1) does not depend on the damping force (i.e, the damping force parameter a). Therefore, the small initial data criterion (theorem 1.1) can be applied in equation (1.2) with any positive damped parameter a.

Remark 1.3. For equation (1.1) (i.e. equation (1.2) with $a = 0$), from Zhang [22], theorem 1.1 is still true.

Remark 1.4. When $N = 2$ and $p = 3$, Fibich[5] used a special transformation $\varphi(t, x) = u(t, x)\,\mathrm{e}^{-at}$ to get a small initial data criterion with L^2-norm of global existence: if the initial data φ_0 satisfies $\|\varphi_0\|_{L^2} < \|Q\|_{L^2}$, then the solutions of the Cauchy problem (1.2)–(1.3) globally exist. In fact, for the case of $p = 1 + \frac{4}{N}$, the above result is also true, which implies theorem 1.1. But, this transformation cannot be used to study the case of $1 + \frac{4}{N} < p < \frac{N+2}{(N-2)^+}$ to obtain a similar result as the case of $p = 1 + \frac{4}{N}$. For the case of $1 + \frac{4}{N} < p < \frac{N+2}{(N-2)^+}$, theorem 1.1 is a sufficient condition of global existence, however, unfortunately, we do not know whether it is optimum.

In this paper, we use $\|\cdot\|_{H^1}$ to denote the norm of $H^1(\mathbf{R}^N)$ and use $\|\cdot\|_{L^p}$ to denote the norm of $L^p(\mathbf{R}^N)$. For simplicity, hereafter, we will denote $\int_{\mathbf{R}^N} \cdot\mathrm{d}x$ by $\int \cdot\mathrm{d}x$ and use C to denote various positive constants. This paper is organized as follows. In the second section, we give some concerned preliminaries. In the third section, we prove theorem 1.1.

2. Preliminaries

First, let $1 < p < \frac{N+2}{(N-2)^+}$ and $u \in H^1(\mathbf{R}^N)$. We define some functionals as follows:

$$E(u) := \frac{1}{2}\int\left[|\nabla u|^2 - \frac{2}{p+1}|u|^{p+1}\right]\mathrm{d}x, \tag{2.1}$$

$$K(u) := \int\left[|\nabla u|^2 - |u|^{p+1}\right]\mathrm{d}x, \tag{2.2}$$

$$L(u) := \frac{1}{2}\int\left[|\nabla u|^2 + |u|^2 - \frac{2}{p+1}|u|^{p+1}\right]\mathrm{d}x \tag{2.3}$$

and

$$R(u) := \int[|\nabla u|^2 + |u|^2 - |u|^{p+1}]\,\mathrm{d}x. \tag{2.4}$$

Then pose a variational problem

$$\begin{cases} d = \inf\limits_{u\in\Omega} L(u), \\ \Omega := \{u \in H^1(\mathbf{R}^N)\backslash\{0\} : R(u) = 0\}. \end{cases} \tag{2.5}$$

Proposition 2.1 [12, 15]**.** *For $1 < p < \frac{N+2}{(N-2)^+}$, let the initial datum $\varphi_0 \in H^1(\mathbf{R}^N)$ and let $\varphi(t)$ be the solution of the Cauchy problem (1.2)–(1.3) on $[0, T_a)$. Then for any $t \in [0, T_a)$,*

$$M(\varphi(t)) := \frac{1}{2}\int|\varphi(t)|^2\,\mathrm{d}x = \mathrm{e}^{-2at}M(\varphi_0), \tag{2.6}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}E(\varphi(t)) = -aK(\varphi(t)). \tag{2.7}$$

Remark 2.1. For equation (1.2), if the damped parameter $a = 0$, the mass and the energy are conservational [4]. More precisely, for all $t \in [0, T_a(\varphi_0))$, $M(\varphi(t)) = M(\varphi_0)$ and $E(\varphi(t)) = E(\varphi_0)$. However, if the damped parameter $a \neq 0$, from proposition 2.1, we know that the mass $M(\varphi(t))$ and the energy $E(\varphi(t))$ lose the conservation laws. This reflects the difference between the damping and the dampingless.

Remark 2.2. From (2.6), we know that equation (1.2) cannot have any stationary or time-periodic solutions. Otherwise, if equation (1.2) had a stationary or time-periodic solution v, then $M(\varphi_0) = \frac{1}{2}\int |\varphi_0|^2\,\mathrm{d}x = \frac{1}{2}\int |v|^2\,\mathrm{d}x = M(\varphi(t))$, which violates (2.6). However, when the damped parameter $a = 0$, equation (1.2) [13] has the standing waves (time-periodic solutions) $\varphi(x, t) = \mathrm{e}^{\mathrm{i}wt}u(x)$, where $w \in \mathbf{R}$ is the frequency and $u(x)$ satisfies the nonlinear elliptic equation

$$-\Delta u + wu - |u|^{p-1}u = 0, \qquad u \in H^1(\mathbf{R}^N).$$

This also reflects the heavy effect of damping on equation (1.2).

Proposition 2.2 [3, 13]**.** *For the variational problem (2.5), assume* $1 < p < \frac{N+2}{(N-2)^+}$. *Then*

$$d = \min_{u\in\Omega} L(u). \tag{2.8}$$

Furthermore, $d > 0$ *and the minimizer u of the variational problem (2.5) is the positive and spherically symmetric solution of equation (1.4).*

Proposition 2.3. *For* $1 < p < \frac{N+2}{(N-2)^+}$, *let* $Q \in H^1(\mathbf{R}^N)$ *be the positive and spherically symmetric solution of equation (1.4). Then the minimizing value d of the variational problem (2.5) satisfies* $d = \frac{p-1}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2$.

Proof. Let $Q \in H^1(\mathbf{R}^N)$ be the positive and spherically symmetric solution of equation (1.4), i.e.

$$-\Delta Q + Q - |Q|^{p-1}Q = 0, \qquad Q \in H^1(\mathbf{R}^N). \tag{2.9}$$

Then, multiplying equation (2.9) with Q and integrating on $\mathbf{R}^N$, one has

$$\int [|\nabla Q|^2 + |Q|^2 - |Q|^{p+1}]\,\mathrm{d}x = 0. \tag{2.10}$$

Also, multiplying equation (2.9) with $x\nabla Q$ and integrating on $\mathbf{R}^N$, one has

$$\int \frac{2-N}{2}|\nabla Q|^2 - \frac{N}{2}|Q|^2 + \frac{N}{p+1}|Q|^{p+1}]\,\mathrm{d}x = 0. \tag{2.11}$$

It follows from (2.10) and (2.11) that

$$\int |Q|^{p+1}\,\mathrm{d}x = \frac{2(p+1)}{2(p+1)-N(p-1)}\int |Q|^2\,\mathrm{d}x. \tag{2.12}$$

On the other hand, from proposition 2.2, we know that Q also is the minimizer of the variational problem (2.5). So

$$d = L(Q) = \frac{1}{2}\int \left[|\nabla Q|^2 + |Q|^2 - \frac{2}{p+1}|Q|^{p+1}\right]\mathrm{d}x, \tag{2.13}$$

and

$$R(Q) = \int [|\nabla Q|^2 + |Q|^2 - |Q|^{p+1}]\,\mathrm{d}x = 0. \tag{2.14}$$

Then from (2.12), (2.13) and (2.14), one has $d = \frac{p-1}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2$. The proof is completed. □

3. Proof of the main result

In this section, we will prove theorem 1.1. To begin with, we establish an invariant evolution flow of the Cauchy problem (1.2)–(1.3). Put

$$K = \left\{ u \in H^1(\mathbf{R}^N) : R(u) > 0 \text{ and } L(u) < \frac{p-1}{2(p+1) - N(p-1)} \|Q\|_{L^2}^2 \right\}, \tag{3.1}$$

where $Q \in H^1(\mathbf{R}^N)$ is the positive and spherically symmetric solution of equation (1.4).

Proposition 3.1. *Assume* $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$. *Then K is invariant under the flow generated by the Cauchy problem (1.2)–(1.3). More precisely, if the initial datum* $\varphi_0 \in K$, *then the solution* $\varphi(t)$ *of the Cauchy problem (1.2)–(1.3) still satisfies* $\varphi(t) \in K$.

Proof. Assume $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$. Let the initial datum $\varphi_0 \in H^1(\mathbf{R}^N)$ and let $\varphi(t)$ denote the solution of the Cauchy problem (1.2)–(1.3) on $t \in [0, T_a)$. Let $Q \in H^1(\mathbf{R}^N)$ be the positive and spherically symmetric solution of equation (1.4). From proposition 2.2, proposition 2.3 and $\varphi_0 \in K$, it follows that $R(\varphi_0) > 0$ and $L(\varphi_0) < d$. At the same time, from (2.6) and (2.7), one has that

$$\frac{\mathrm{d}}{\mathrm{d}t} L(\varphi(t)) = -aR(\varphi(t)). \tag{3.2}$$

In the following, we use the contradiction method to prove that $\varphi(t) \in K$.

If $\varphi(t) \notin K$, by $\varphi_0 \in K$ and the continuity with respect to t, there would exist a $t_1 \in [0, T_a)$ such that $\varphi(t) \in K$ for all $t \in [0, t_1)$ and $\varphi(t_1) \notin K$. Then, $R(\varphi(t)) \geqslant 0$ for all $t \in [0, t_1]$. From (3.2), it follows that

$$L(\varphi(t_1)) \leqslant L(\varphi_0) < d. \tag{3.3}$$

Since $\varphi(t_1) \notin K$, then $R(\varphi(t_1)) = 0$. Then from the variational problem (2.5), it follows that $L(\varphi(t_1)) \geqslant d$, which violates (3.3). Therefore $\varphi(t) \in K$. The proof is completed. □

Remark 3.1. For equation (1.2), if the damped parameter $a = 0$, then the sets

$$K = \left\{ u \in H : R(u) > 0 \text{ and } L(u) < \frac{p-1}{2(p+1) - N(p-1)} \|Q\|_{L^2}^2 \right\},$$

and

$$K' = \left\{ u \in H : R(u) < 0 \text{ and } L(u) < \frac{p-1}{2(p+1) - N(p-1)} \|Q\|_{L^2}^2 \right\}$$

are all invariant under the flow generated by the Cauchy problem (1.2)–(1.3) [21, 22]. If the damped parameter $a \neq 0$, from proposition 3.1, we only know that K is invariant. However, we do not know whether K' is invariant with the damped parameter $a \neq 0$. This also reflects the difference between the damping and the dampingless.

Proposition 3.2. *Assume* $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$. *Let the initial data* $\varphi_0 \in K$. *Then the solution* $\varphi(t)$ *of the Cauchy problem (1.2)–(1.3) globally exists in* $H^1(\mathbf{R}^N)$.

Proof. For $1 + \frac{4}{N} \leqslant p < \frac{N+2}{(N-2)^+}$, let $\varphi(t)$ be the solution of the Cauchy problem (1.2)–(1.3) with the initial data $\varphi_0 \in K$. Then from proposition 3.1, it follows that $\varphi(t) \in K$. In other words, there have $L(\varphi(t)) < d$ and $R(\varphi(t)) > 0$, which implies that $\varphi(t)$ is bounded in $H^1(\mathbf{R}^N)$. Therefore, $\varphi(t)$ globally exists in $H^1(\mathbf{R}^N)$. The proof is completed. □

Proof of theorem 1.1. Let $\varphi_0(x) \in H^1(\mathbf{R}^N)\backslash\{0\}$ satisfy $\|\varphi_0\|_{H^1} \leqslant \sqrt{\frac{2(p-1)}{2(p+1)-N(p-1)}}\|Q\|_{L^2}$, where Q is the positive and spherically symmetric solution of equation (1.4). It follows from proposition 2.3 that

$$\begin{aligned}L(\varphi_0) &= \frac{1}{2}\int[|\nabla\varphi_0|^2 + |\varphi_0|^2 - \frac{2}{p+1}|\varphi_0|^{p+1}]\,\mathrm{d}x\\ &< \frac{1}{2}\int[|\nabla\varphi_0|^2 + |\varphi_0|^2]\,\mathrm{d}x\\ &\leqslant \frac{1}{2}\cdot\frac{2(p-1)}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2\\ &= \frac{p-1}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2\\ &= d.\end{aligned} \tag{3.4}$$

Therefore, to prove theorem 1.1, from propositions 3.1 and 3.2, we only need to prove that φ_0 also satisfies $R(\varphi_0) > 0$. We use the contradiction method to prove it.

If $R(\varphi_0) > 0$ were not true, one would have

$$R(\varphi_0) = \int[|\nabla\varphi_0|^2 + |\varphi_0|^2 - |\varphi_0|^{p+1}]\,\mathrm{d}x \leqslant 0. \tag{3.5}$$

First, we prove $R(\varphi_0) \neq 0$. Otherwise, $\varphi_0 \in \Omega$. Then it follows from the variational problem (2.5) that $L(\varphi_0) \geqslant d$, which violates (3.4). Then

$$R(\varphi_0) = \int[|\nabla\varphi_0|^2 + |\varphi_0|^2 - |\varphi_0|^{p+1}]\,\mathrm{d}x < 0. \tag{3.6}$$

Thus there exists a $\mu \in (0, 1)$ such that $R(\mu\varphi_0) = 0$, which implies that $\mu\varphi_0 \in \Omega$. Then it also follows from the variational problem (2.5) that $L(\mu\varphi_0) \geqslant d$. But

$$\begin{aligned}L(\mu\varphi_0) &= \frac{1}{2}\mu^2\int[|\nabla\varphi_0|^2 + |\varphi_0|^2]\,\mathrm{d}x - \frac{1}{p+1}\mu^{p+1}\int|\varphi_0|^{p+1}\,\mathrm{d}x\\ &< \frac{1}{2}\mu^2\int[|\nabla\varphi_0|^2 + |\varphi_0|^2]\,\mathrm{d}x\\ &\leqslant \frac{1}{2}\mu^2\cdot\frac{2(p-1)}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2\\ &= \mu^2\frac{p-1}{2(p+1)-N(p-1)}\|Q\|_{L^2}^2\\ &= \mu d\\ &< d.\end{aligned} \tag{3.7}$$

This is a contradiction. Therefore $\varphi_0 \in K$. By proposition 3.2, we obtain the solution $\varphi(t)$ of the Cauchy problem (1.2)–(1.3) globally exists in $H^1(\mathbf{R}^N)$. □

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